5/18/18 "Live as if you were to die tomorrow. Learn as if you were to live forever."
-Mahatma Gandhi

HW: "2017 A2 CC L79 Normal Distribution 2 HW"
Test 2 Wednesday 5/30

AIM: What is a Z-Score?

Warm Up:

NAME_____

A2CC: THE NORMAL DISTRIBUTION -- HOMEWORK

1. A variable is normally distributed with a mean of 16 and a standard deviation of 6. Find the percent of the data set that:

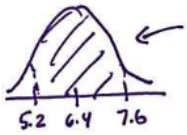
   (a) is greater than 16 50% (b) falls between 10 and 22 68.2% (c) is greater than 28 2.3%

   (d) is less than 1 .6% (e) falls between 4 and 19 66.8% (f) falls between 22 and 31 15.3%

2. The weights of Siamese cats are normally distributed with a mean of 6.4 pounds and a standard deviation of 0.8 pounds. If a breeder of Siamese cats has 128 in his care, how many can he expect to have weights between 5.2 and 7.6 pounds?

   (1) 106 (3) 98

   (2) 49 (4) 111

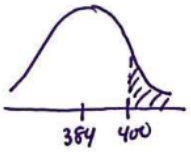


$\mu = 6.4$
$\sigma = .8$

3. A recent study found that the mean amount spent by individuals on a music service website was normally distributed with a mean of \$384 with a standard deviation of \$48. Which of the following gives the proportion of the individuals that spend more than \$400?

   (1) 0.43 (3) 0.12

   (2) 0.74 (4) 0.37

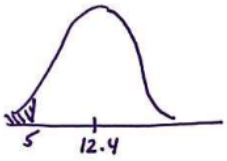


$\mu = 384$
$\sigma = 48$

4. The hold time experienced by people calling a government agency was found to be normally distributed with a mean of 12.4 minutes and a standard deviation of 4.3 minutes. Which percent below represents the percent of calls answered in less than 5 minutes?

   (1) 4.3% (3) 6.8%

   (2) 5.3% (4) 12.9%

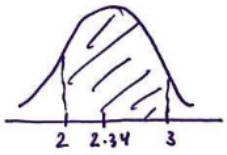


$\mu = 12.4$
$\sigma = 4.3$

5. The national average price per gallon for gasoline is normally distributed with a mean (currently) of \$2.34 per gallon with a standard deviation of \$0.26 per gallon. Which of the following represents the proportion of the gas prices that lie between \$2.00 and \$3.00?

   (1) 56% (3) 84%

   (2) 72% (4) 90%

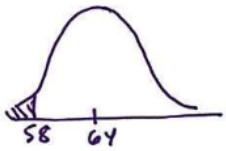


$\mu = 2.34$
$\sigma = .26$

6. If one quart bottles of apple juice have weights that are normally distributed with a mean of 64 ounces and a standard deviation of 3 ounces, what percent of bottles would be expected to have less than 58 ounces?

(1) 6.7% (3) 0.6%

(2) 15.0% (4) 2.3%

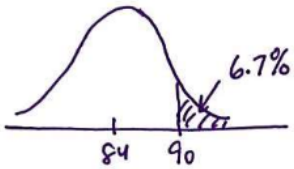


$\mu = 64$
$\sigma = 3$

7. Historically daily high temperatures in July in Red Hook, New York, are normally distributed with a mean of 84°F and a standard deviation of 4°F. How many of the 31 days of July can a person expect to have temperatures above 90°F?

(1) 6 (3) 9

(2) 2 (4) 4



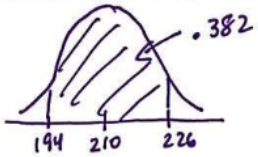
$\mu = 84$
$\sigma = 4$

$(.067)(31)$

8. The lengths of songs on the radio are normally distributed with a mean length of 210 seconds. If 38.2% of all songs have lengths between 194 and 226 seconds, then the standard deviation of this distribution is

(1) 16 seconds (3) 8 seconds

(2) 32 seconds (4) 64 seconds

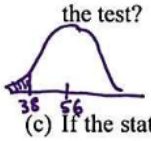


$\mu = 210$

Try normalcdf with each σ to find the correct one.

9. On a recent statewide math test, the raw score average was 56 points with a standard deviation of 18. If the scores were normally distributed and 24,000 students took the test, answer the following questions.

(a) What percent of students scored below a 38 on the test?



15.9%

(b) How many students scored less than a 38?

$(.158655\,2596)(24000) = 3807.7$

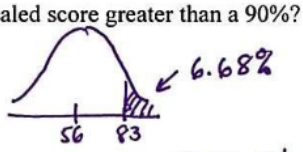
3,808 students

(c) If the state would like to scale the test so that a 90% would correspond to a raw score that is one and a half standard deviations above the mean, what raw score is needed for a 90%?

$\mu + 1.5\sigma$

$56 + 1.5(18) = 83$

(d) How many of the 24,000 students receive a scaled score greater than a 90%?



1603 students

How many std. deviations away from the mean

THE Z-SCORE OF A DATA VALUE

For a data point x_i, its z-score is calculated by: $z = \frac{x_i - \mu}{\sigma}$. It calculates how far from the mean, in terms of standard deviations, a data point lies. It can be positive if the data point lies above the mean or negative if the data point lies below the mean.

$$\frac{\text{your score} - \text{mean score}}{\text{Std. deviation}}$$

***Exercise* #1:** Boy's heights in seventh grade are normally distributed with a mean height of 62 inches and a standard deviation of 3.2 inches. Find z-scores, rounded to the nearest hundredth, for each of the following heights. Show the calculation that leads to your answer.

(a) $x_i = 66$ inches

$$z = \frac{66 - 62}{3.2} = 1.25$$

(b) $x_i = 57$ inches

$$z = \frac{57 - 62}{3.2} = -1.56$$

Z-scores give us a way to compare how far a data point is away from its mean in terms of standard deviations. We should be able to compute a z-score for a data value and go in the opposite direction.

***Exercise* #2:** Jeremiah took a standardized test where the mean score was a 560 and the standard deviation was 45. If Jeremiah's score resulted in a z-value of 1.84, then what was Jeremiah's score to the nearest whole number?

$$1.84 = \frac{x - 560}{45}$$

$$82.8 = x - 560$$

$$+560 \qquad +560$$

$$642.8 = x$$

Jeremiah scored 642.8

***Exercise* #3:** The lengths of full grown sockeye salmon are normally distributed with a mean of 29.2 inches and a standard deviation of 2.4 inches.

(a) Find z-scores for sockeye salmon whose lengths are 25 inches to 32 inches. Round to the nearest hundredth.

$x = 25$

$$z = \frac{25 - 29.2}{2.4} = -1.75$$

$x = 32$

$$z = \frac{32 - 29.2}{2.4} = 1.17$$

(b) Determine the proportion of the sockeye salmon population, to the nearest percent, that lies between 25 inches and 32 inches. Illustrate your work graphically.

lower = 25
upper = 32
$\mu = 29.2$
$\theta = 2.4$

84%

lower = -1.75
upper = 1.17
$\mu = 0$
$\theta = 1$

84%

(*) If we use z-scores then $\mu = 0$ $\theta = 1$ Always

***Exercise* #4:** If the scores on a standardized test are normally distributed with a mean of 560 and a standard deviation of 75. Answer the following questions by using z-scores and illustrating with a picture.

(a) Find the probability that a test picked at random would have a score larger than 720. Round to the nearest tenth of a percent.

lower: 720
upper: 10000
$\mu = 560$
$\theta = 75$

$\approx$ 1.6%

(b) Find the probability that a completed test picked at random would have a score less than 500. Round to the nearest tenth of a percent.

lower: -10000
upper: 500
$\mu = 560$
$\theta = 75$

$\approx$ 21.2%

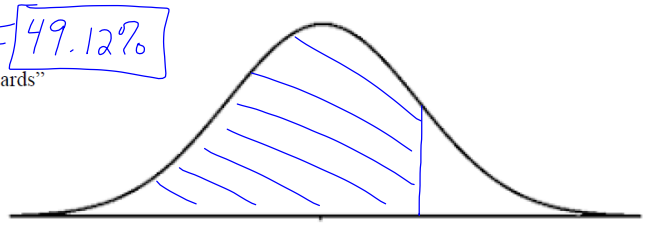
(c) Find the probability that a completed test picked at random would have a score between 500 and 600.

lower: 500 $\theta = 75$
upper: 600
$\mu = 560$

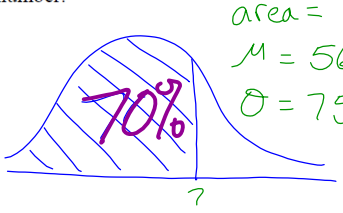
$\approx$ 49.12%

Using the calculator to "work backwards"

"Inv Norm" to find value given %

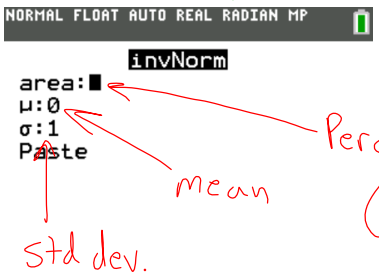


(d) What is the lowest score a student can get and still score greater than 70% of students? Illustrate with a picture. Round to nearest whole number.



area = .7
$\mu = 560$
$\theta = 75$

$\approx$ 599

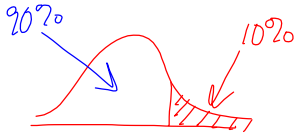


Percent as a decimal (area to left)

mean

std dev.

(e) Ten percent of test scores would be greater than what test score? Illustrate with a picture. Round to nearest whole number.



Think what's left?

Area = .9
$\mu = 560$
$\theta = 75$

$\approx$ 656

This process is sometimes used to determine a particular data point's **percentile**, which is the **percent of the population less than or equal to the data point**.

***Exercise* #5:** The average weight of full grown beef cows is 1470 pounds with a standard deviation of 230 pounds. If the weights are normally distributed, what is the percentile rank of a cow that weighs 1,750 pounds?

(1) 89^{th} (3) 49^{th}

(2) 76^{th} (4) 35^{th}

***Exercise* #6:** Using the information given in exercise #5, what would the weight of a cow be whose weight is at the 68^{th} percentile?